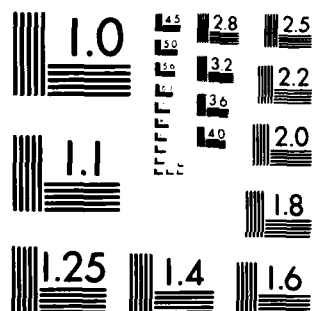END
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PREDICTION OF WAVE HEIGHT OF WIND-GENERATED SEAS IN FINITE WATER DEPTH

BY

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AND

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SEPTEMBER 1984

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This paper presents a method to statistically predict peaks, troughs, and peak-to-trough excursions of waves in finite water depth such as waves in harbors, bays, and near-shore and offshore seas. The probability density functions are developed based on the concept that waves in finite water depth are a non-Gaussian random process with parameters which depends on the sea severity and water depth.

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ABSTRACT

This paper presents a method to statistically predict peaks, troughs, and peak-to-trough excursions of waves in finite water depth such as waves in harbors, bays, and near-shore and offshore seas. The probability density functions are developed based on the concept that waves in finite water depth are a non-Gaussian random process with parameters which depends on the sea severity and water depth. As an example of application of the present method, numerical computations are carried out for water depth of 8.8 m (28.8 ft) in a sea of significant wave height 3.2 m (10.4 ft). The results of computation show that the newly-developed probability density functions of peaks and troughs both reasonably agree with the histograms constructed from measured data obtained during the ARSLOE Project.

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INTRODUCTION

In the past two decades considerable attention has been given to statistical prediction of wind-generated wave characteristics of random seas in the ocean; however, relatively little is known of wave characteristics in finite water depth such as harbors, bays, etc. In general, wave time histories observed in finite water depth areas show a definite excess of high crests and shallow troughs as demonstrated in Figure 1(a) in contrast to wave profiles observed in deep water (see Figure 1b). The skewness of wave profiles observed in finite water depths is associated with the non-linear characteristics of the waves, and it depends on water depth and sea severity.

From the stochastic viewpoint, waves in finite water depth cannot be considered as a Gaussian random process. Therefore, currently available methods for predicting wave heights based on the concept of a Gaussian random process (which is applicable for predicting ocean wave heights) is no longer valid for probabilistic prediction of wave heights in finite water depth. The results of analysis carried out on wave data obtained during a storm at Duck, North Carolina, for water depths ranging from 1.4 m (4.4 ft) to 24.4 m (80.0 ft) have indicated that the probability distribution of the wave profile deviates substantially from the normal (Gaussian) probability distribution, and that the probability distribution of wave heights (peak-to-trough excursions) does not follow the Rayleigh probability law even for waves measured at a relatively deep location, 24.4 m when the sea state becomes severe (Ochi et al. 1982).

Hence, it is highly desirable to develop a method for predicting heights of waves in finite water depth based on the concept of a non-Gaussian random process. It is also noted that the magnitude of peaks of waves in finite

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water depth is much greater than that of troughs, in general. Therefore, for waves in finite water depth, probabilistic characteristics of peaks and troughs should be evaluated independently in contrast to the prediction method for ocean waves in which probabilistic characteristics of peaks and troughs are assumed to be the same.

Several studies have been carried out recently on the probability distribution of wave heights for waves in finite water depth. Thompson (1980), Arhan and Plaisted (1981), Tayfun (1980) (1983), Thornton and Guza (1983), and Huang et al. (1983) developed methods for predicting wave heights. However, the probability density functions derived in these studies are not based on the non-Gaussian random process concept; instead, the probability distributions are developed based on some assumptions. For example, waves are expressed as a Stokes expansion to the second and third order components. On the other hand, Bitner (1980) developed a probability density function applicable for wave heights based on the non-Gaussian random process concept. In her development, however, peaks and troughs are assumed to be equal, and the probability density function is not given as a function of water depth.

The purpose of the present study is to develop a method to statistically predict peaks, troughs, and peak-to-trough excursions of waves in finite water depth such as waves in harbors and bays, and near-shore and offshore seas. The development of the probability density function necessary for prediction is based on the concept that waves in finite water depth are a non-Gaussian random process with parameters which depend on the sea severity and water depth. That is, the limiting sea severity (significant wave height) above which the waves are considered to be a non-Gaussian random process is first evaluated as a function of water depth. Then, the magnitude of peaks (or

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troughs, or peak-to-trough excursions) of waves categorized as a non-Gaussian random process can be predicted by applying a newly-developed probability density function assuming that the wave spectrum is narrow-banded.

The paper consists of three sections. The first section outlines the probability function applicable to a non-Gaussian random process originally developed by Longuet-Higgins (1963). From analysis of more than 500 wave records measured at various water depths during the growing stage of storm in the ARSLOE Project, it was found that two parameters involved in the Longuet-Higgins probability density function are significant for predicting wave characteristics in finite water depth, and these two parameters are expressed as a function of sea severity (significant wave height) and water depth. The second section discusses the method for predicting peaks and troughs as well as peak-to-trough excursions of waves in finite water depth developed based on the non-Gaussian probability function, and the third section presents the application of the prediction method developed in the present study for estimating peaks, troughs, and peak-to-trough excursions of waves in a given sea severity (significant wave height) at a specified water depth. Numerical computations are carried out for water depths of 8.8 m (28.8 ft) and the results are compared with data measured during the ARSLOE Project.

PROBABILITY FUNCTION OF NON-GAUSSIAN RANDOM PROCESSES

As stated in the Introduction, wind-generated random waves in finite water depths are considered to be a non-Gaussian random process. The

probability functions representing a non-Gaussian random process have been developed from two different approaches; one based on the probability theory (Gram-Charlier series, see Cramer 1946, Edgeworth 1905, and Longuet-Higgins 1963), the other based on the approximation that waves are expressed as a Stokes expansion to the 2nd and 3rd components (Tayfun 1980, Huang and Long 1980, and Huang et al. 1983). The former approach does not include any assumption and all probability distribution functions are given in the form of a series. These probability functions are similar; in particular, the probability functions derived by Edgeworth and Longuet-Higgins are the same and they appear to be more accurate than that derived by Gram-Charlier. Hence, in the present study, the non-Gaussian probability density function derived by Longuet-Higgins will be applied in analyzing wave data obtained in finite water depths.

Let the wave deviation from the mean (namely, wave profile) be X. The probability density function of X is given as follows (Longuet-Higgins 1963):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \left[1 + \frac{\lambda_3}{3!} H_3\left(\frac{x}{\sigma}\right) + \frac{\lambda_4}{4!} H_4\left(\frac{x}{\sigma}\right) + \frac{\lambda_5}{5!} H_5\left(\frac{x}{\sigma}\right) \right.$$
$$\left. + \frac{\lambda_3^2}{2! \, (3!)^2} H_6\left(\frac{x}{\sigma}\right) + \frac{2\lambda_3 \, \lambda_4}{2! \, (3!) \, (4!)} H_7\left(\frac{x}{\sigma}\right) + \cdots \right]$$

$$-\infty < x < \infty \qquad (1)$$

where,
$$\sigma^2 = \text{variance of } x$$
$$\lambda_3 = \kappa_3/(\kappa_2)^{1.5}$$
$$\lambda_4 = \kappa_4/\kappa_2^2$$
$$\lambda_5 = \kappa_5/(\kappa_2)^{2.5}$$

$$k_j = \text{cumulants}$$

$$k_2 = m_2 - m_1^2 = \text{variance}$$

$$k_3 = m_3 - 3m_2m_1 + 2m_1^3$$

$$k_4 = m_4 - 3m_2^2 - 4m_3m_1 + 12m_2m_1^2 - 6m_1^4$$

$$k_5 = m_5 - 5m_4m_1 - 10m_3m_2 + 20m_3m_1^2$$
$$+ 30m_2^2m_1 - 60m_2m_1^3 + 24m_1^5$$

$$m_j = \text{j-th moment of the probability density function}$$

$$H_j\left(\frac{x}{\sigma}\right) = \text{Hermite polynomial of degree } j$$

Note that λ_3 is called the skewness of the random variable X and λ is equal to the kurtosis minus 3. The values of λ_3 and λ_4 may be positi i s well as negative in general; however, positive λ_3 are observed for most wave profiles in finite water depths.

The following remarks are made on the probability density function given in Equation (1):

(1) The probability density function f(x) at times becomes negative for large negative x-values depending on σ-values. However, this should not cause any serious trouble, in practice, since the x-values where f(x) becomes negative are usually outside the range of x where the histogram exists as will be shown later.

(2) Generally speaking, the accuracy of a function which is expressed in the form of a series increases with increase in higher order terms. However, this is not the case for the probability density function given in Equation (1). The results of comparison between histograms and the probability density function have shown that higher order terms do not necessarily yield better agreement (Honda and Mitsuyasu 1976, Ochi et al. 1982). Therefore, it is highly desirable to examine terms of the series which significantly contribute to the distribution.

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To substantiate the statement given above, Figure 2 was prepared. The figure shows an example of a comparison of the probability density function given in Equation (1) with a histogram of the wave profile obtained at a water depth of 1.35 m (4.43 ft) in a severe storm of significant wave height of 2.05 m (6.72 ft). The data were obtained during a storm in the ARSLOE Project by the Coastal Engineering Research Center at Duck, North Carolina. The example shown in Figure 2 was chosen since the contribution of each parameter, λ_3, λ_4, etc., is pronounced because of the extremely shallow water depth.

It can be seen in Figure 2 that,

(1) The histogram deviates substantially from the normal probability distribution.

(2) The probability density function computed by taking various terms of Equation (1) become negative for large negative x. However, the magnitude of the negative probability density is relatively small, on the order of less than two percent. Furthermore, the negative probability density occurs outside range of the histogram. Hence, it will not cause any serious problem in practice if we assume this negative probability density to be zero, and, in turn, the entire probability density function is normalized such that the area of the density function becomes unity.

(3) The probability density function representing the first two terms which includes the parameter λ_3 agrees reasonably well with the histogram.

(4) The agreement with the histogram becomes poor if the term with the parameter $\lambda_3{}^2$ is included in addition to the λ_3-term in the probability density function. Although it is not shown in the figure, the same trend is obtained if the term with the parameter λ_4 is included in addition to the λ_3-term in the probability density function.

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(5) The probability density function consisting of terms with the parameters λ_3, λ_3^2, and λ_4 agrees well with the histogram.

(6) The addition of the parameter λ_5 to the probability density function does not yield any appreciable change in the shape of the probability density function.

The same trend was observed for many other wave records obtained at various water depths in various sea severities (Ochi and Wang 1984); hence, we may safely conclude that the parameter λ_3 which represents the skewness of the wave profile is the dominant parameter affecting the non-Gaussian characteristics of waves in finite water depth and that the combination of the parameters λ_3, λ_3^2, and λ_4 best represents the non-Gaussian probability density function given in Equation (1). The combination of the parameters λ_3 and λ_3^2 or the combination of the parameters λ_3 and λ_4 do not yield satisfactory agreement between the histogram and the non-Gaussian probability density function.

Based on the above-mentioned conclusion, only the two parameters λ_3 and λ_4 involved in the non-Gaussian probability density function are considered in predicting heights (peak-to-trough excursions) of waves in finite water depth.

In general, the non-Gaussian characteristics of wind-generated waves depends not only on water depth but also sea severity. Even though the water depth is finite, waves may be considered to be a Gaussian random process if the sea state is mild. Therefore, for a given water depth, it may be of considerable interest to examine the limiting sea severity below which wind-generated waves are considered to be Gaussian. For this, non-Gaussian analyses were carried out on more than 500 wave records obtained during a storm in the ARSLOE Project, and the formulae for evaluating the parameters λ_3 and λ_4 were derived as a function of water depth and sea severity (Ochi and Wang 1984). That is, the parameter λ_3 can be evaluated by,

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$$\lambda_3 = 1.10 \ e^{-0.39h} \cdot H_s^{0.74} \ h^{0.62} \tag{2}$$

where, h = water depth in meters

                           H_s = significant wave height in meters

Furthermore, wave records as well as histograms of the wave profile were examined to find the minimum significant wave height above which the non-Gaussian characteristics were observed. The results are shown in Figure 3 together with the curves indicating various λ_3-values as a function of water depth computed by Equation (2). As can be seen in the figure, the minimum significant wave height above which non-Gaussian characteristics are observed agrees well with the curve for which the parameter λ_3 is equal to 0.2. Hence, we may conclude that waves in mild seas for which the parameter λ_3 is less than 0.2 can be considered as a Gaussian random process.

It was also found from the analysis that the parameter λ_4 can be expressed approximately as a function of λ_3 as shown in Figure 4. The average curve drawn in the figure is expressed by the following formulae:

$$\lambda_4 = \begin{cases} -0.15 + 1.10 \ (\lambda_3 - 0.20)^{1.17} & \text{for } 0.2 < \lambda < 0.5 \\ \\ -0.15 + 1.10 \ (\lambda_3 - 0.20)^{1.17} + 1.48 \ (\lambda_3 - 0.50)^{1.47} \\ & \text{for } \lambda_3 > 0.5 \end{cases} \tag{3}$$

In summary, waves for a given water depth may be considered as a Gaussian random process in mild sea conditions if the parameter λ_3 evaluated by applying Equation (2) is less than 0.2; however, in severe seas for which the

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parameter λ_3 is greater than 0.2, waves must be considered as a non-Gaussian random process represented by Equation (1) consisting of terms containing the parameters λ_3, λ_3^2, and λ_4. The parameter λ_4 can be evaluated by Equation (3).

PROBABILITY FUNCTIONS OF PEAKS AND TROUGHS

One of the most distinct statistical characteristics of waves in finite water depths is that the wave profile does not follow a Gaussian distribution as discussed in the previous section; and hence, the statistical properties of the positive part of the wave profile are different from those of the negative part. This implies that the probability functions applicable for peaks and troughs are different and should be derived independently. We first consider the probability density function for the peaks.

Let us assume that a wave spectrum is narrow-banded and hence there exists only a single peak during every half-cycle period. Then, the probability that the peak, which is a random variable denoted by Ξ_+ , exceeds a specified level ξ_+ is given as the expected value of the ratio of number of peaks exceeding the level ξ to the number of peaks on the positive domain. That is,

$$P_r\{\Xi_+ > \xi_+\} \;=\; 1 - F(\xi_+) \;=\; E\left[\frac{N_{\xi+}}{N_+}\right] \sim \frac{\bar{N}_{\xi+}}{\bar{N}_+} \tag{4}$$

where, $F(\xi_+)$ = cumulative distribution function of Ξ_+

$N_{\xi+}$ = number of peaks exceeding a level ξ_+ per unit time

N_+ = number of peaks on the positive domain per unit time

$\bar{N}_{\xi+}$ = expected value of $N_{\xi+}$

$\bar{N}_+$ = expected value of N_+

It is noted that the expected value $E[N_{\xi+}/N_+]$ is equal to the ratio of the individual expected values, denoted by $\bar{N}_{\xi+}$ and $\bar{N}_+$, respectively, only if we assume that the random variables $(N_{\xi+}/N_+)$ and N_+ are statistically independent. This assumption, however, appears to be acceptable for wind-generated waves, in general. The expected values $\bar{N}_{\xi+}$ and $\bar{N}_+$ can be evaluated from the joint probability density function of displacement and velocity as follows:

$$\bar{N}_{\xi+} = \int_{-\infty}^{\infty} |\dot{x}| \, f(\xi_+, \dot{x}) \, d\dot{x} \tag{5}$$

$$\bar{N}_+ = \int_{-\infty}^{\infty} |\dot{x}| \, f(0, \dot{x}) \, d\dot{x}$$

From (4) and (5), the probability density function ξ_+ can be derived as,

$$f(\xi_+) = \frac{-\dfrac{d}{d\xi_+} \displaystyle\int_{-\infty}^{\infty} |\dot{x}| \, f(\xi_+, \dot{x}) \, d\dot{x}}{\displaystyle\int_{-\infty}^{\infty} |\dot{x}| \, f(0, \dot{x}) \, d\dot{x}} \tag{6}$$

In the joint probability density function of displacement x and velocity $\dot{x}$, it is assumed that,

(i) The displacement x has a non-Gaussian probability distribution given in Equation (1) which is the product of a Gaussian distribution and a series

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consisting of parameters λ_3, λ_4, etc. which are associated with cumulants. From the results of analysis on the effect of parameters on the probability density function presented in the previous section, the terms with parameters λ_3, $\lambda_3{}^2$, and λ_4 in the series involved in the equation are considered in the following analysis.

(ii) The velocity $\dot{x}$ has approximately a Gaussian distribution with zero mean and variance $\sigma_{\dot{x}}^2$. Results of statistical analysis of velocity obtained from differentiating measured wave profile have indicated that the velocity obeys approximately the Gaussian distribution. Examples showing comparisons between the histogram and the Gaussian distribution of velocities are shown in Figure 3. The figure pertains to waves at a water depth of 3.70 m (12.1 ft) in a sea of significant wave height 2.45 m (8.04 ft).

(iii) The displacement and velocity are statistically independent. The results of analysis carried out on measured wave profiles and velocities obtained by differentiating the profile have shown that the magnitude of the correlation coefficient is small (on the order of 0.2 or less) irrespective of water depth and sea severity.

Taking these three conditions into consideration, the probability density function of peaks can be obtained from Equations (5) and (6) as follows:

$$
f(\xi_+) = \frac{1}{L}\left[\frac{\xi_+}{\sigma_x^2} e^{-\frac{1}{2}\left(\frac{\xi_+}{\sigma_x}\right)^2} \left\{ 1 + \frac{\lambda_3}{3!} H_3\left(\frac{\xi_+}{\sigma_x}\right) + \frac{\lambda_4}{4!} H_4\left(\frac{\xi_+}{\sigma_x}\right) + \frac{\lambda_3^2}{72} H_6\left(\frac{\xi_+}{\sigma_x}\right) \right\} \right.
$$
$$
\left. - e^{-\frac{1}{2}\left(\frac{\xi_+}{\sigma_x}\right)^2} \left(\frac{1}{\sigma_x}\right) \left\{ \frac{\lambda_3}{2!} H_2\left(\frac{\xi_+}{\sigma_x}\right) + \frac{\lambda_4}{3!} H_3\left(\frac{\xi_+}{\sigma_x}\right) + \frac{\lambda_3^2}{12} H_5\left(\frac{\xi_+}{\sigma_x}\right) \right\} \right]
\qquad (7)
$$

$$
0 < \xi_+ < \infty
$$

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where, $L = 1 + \frac{\lambda_4}{3!} - \frac{5}{24} \lambda_3^2$

The above probability density function reduces to the well-known Rayleigh probability density function if the parameters λ_3 and λ_4 are zero which represents a Gaussian random process. It is noted that in order to let the probability density function be zero for $\xi = 0$, the Hermite polynomial $H_2(\frac{\xi_+}{\sigma_x})$ is modified such that it becomes zero for small ξ_+. That is,

$$H_2\left(\frac{\xi_+}{\sigma_x}\right) = \begin{cases} \left(\frac{\xi_+}{\sigma_x}\right)^2 - 1 & \text{for } \frac{\xi}{\sigma_x} > 1 \\ 0 & \text{for } \frac{\xi}{\sigma_x} < 1 \end{cases} \tag{8}$$

For the probability density function specifically applicable to peaks, it is necessary to consider the variance which is affiliated with the positive part (upper portion) of waves, denoted by σ_{x+}^2. This can be evaluated by taking the second moment of the probability density function of the wave profile, $f(x)$, truncated at $x = 0$. That is,

$$\sigma_{x+}^2 = \frac{\int_o^\infty x^2 f(x)\, dx}{\int_o^\infty f(x)\, dx} \tag{9}$$

By applying the probability density function given in Equation (1), the variance σ_{x+}^2 is given as a function of the parameter λ_3. That is,

$$\sigma_{x+}^2 = \left(\frac{6\sqrt{\pi} + 2\sqrt{2}\,\lambda_3}{6\sqrt{\pi} - 2\sqrt{2}\,\lambda_3}\right) \sigma_x^2 \tag{10}$$

The probability density function of troughs of a non-Gaussian random process can be derived through the same procedure as is used for the probability density function of peaks. The form of the probability density function of peaks is the same as given in Equation (7) except ξ should be negative and the variance σ_{x-}^2 applicable for the negative part (lower portion) of waves is given by,

$$\sigma_{x-}^2 = \frac{\int_{-\infty}^{0} x^2 f(x) \, dx}{\int_{-\infty}^{0} f(x) \, dx} = \left(\frac{6\sqrt{\pi} - 2\sqrt{2} \, \lambda_3}{6\sqrt{\pi} + \sqrt{2} \, \lambda_3} \right) \sigma_x^2 \qquad (11)$$

Next, we evaluate the peak-to-trough excursions that wave peaks, ξ_+, and troughs, ξ_-, are statistically independent. By denoting the probability density functions of peaks and troughs by $f_{\xi_+}(\xi_+)$ and $f_{\xi_-}(\xi_-)$, respectively, the probability density function of peak-to-trough excursions can be obtained by the following convolution integral:

$$f(\xi) = \frac{1}{K} \int_0^{\infty} f_{\xi_+}(\xi_+) \cdot f_{\xi_-}(\xi - \xi_+) \, d\xi_+ \qquad 0 < \xi < \infty \qquad (12)$$

where, K is a constant associated with the normalization and it is given by,

$$K = \int_0^{\infty} f(\xi) \, d\xi \qquad (13)$$

The significant wave height and extreme wave height can now be evaluated from Equation (12). That is, the significant wave height, denoted by H_s, is given by,

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$$H_s = \frac{1}{3} \int_{\xi_*}^{\infty} \xi \cdot f(\xi) \, d\xi \qquad (14)$$

where, ξ_* is the value which satisfies the following relationship:

$$\int_{o}^{\xi_*} f(\xi) \, d\xi = \frac{2}{3} \qquad (15)$$

The probable extreme wave height in n-waves, denoted by $\bar{\xi}$, which is the modal value of the n-th order probability density function is given as the value which satisfies the following equation:

$$\int_{o}^{\bar{\xi}} f(\xi) \, d\xi = 1 - \frac{1}{n} \qquad (16)$$

EXAMPLE OF APPLICATION

Prior to presenting an example of application of the method for predicting peaks, troughs, and peak-to-trough excursions (including extreme values) of waves in finite water depth, it may be well to summarize the computation procedure in the following:

Suppose we want to predict extreme wave heights in a sea of significant wave height, H_s (in meters), at a location where the water depth is known as h (in meters). First, an evaluation is made to identify whether or not waves in the specified sea can be considered as a Gaussian random process at this location. This can be done either by computing the skewness λ_3 by Equation (2) or by referring to the limiting line for the Gaussian random process given in Figure 3. If λ_3 is less than or equal to 0.2, then the sea can be

considered to be a Gaussian random process, and hence the prediction can be made by applying the commonly known method for evaluating wave heights in deep water.

If the sea is considered to be a non-Gaussian random process, then the following iteration method is used:

- Evaluate λ_4 from Equation (3).

- An as initial value of the variance, compute $\sigma^2 = (H_s/4)^2$ and evaluate σ_{x+}^2 and σ_{x-}^2 by Equations (10) and (11), respectively.

- Compute the probability density function $f(\xi_+)$ and $f(\xi_-)$ by Equation (7) with σ_3, and σ_4, and σ_{x+}^2 or σ_{x-}^2, respectively.

- Compute the probability density function of the peak-to-trough excursions by Equation (12), and then evaluate the significant wave height by Equation (14).

- Compare the computed significant wave height with the specified value, H_s. Iterate the procedure until these two values agree.

- Significant value and extreme value for the peaks and troughs are evaluated from the final probability density functions.

The iteration method is required, since the relationship between the significant wave height and variance, σ_x^2, which is necessary for computing the probability density function is not known for waves in finite water depth. By applying the iteration method, the variance can be evaluated.

As an example of application, computations are carried out for waves at a water depth of 8.8 m (28.8 ft) in a sea of significant wave height 3.18 m (10.4 ft), and the results are compared with the measured data obtained during the ARSLOE Project as shown in Figures 6 through 8. Included also in these figures is the Rayleigh probability distribution which is commonly used for

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predicting waves in deep water. As can be seen in Figure 6 which shows a comparison of peak values, the probability density function computed by Equation (7) reasonably agrees with the histogram constructed from measured data, while the Rayleigh probability density function substantially underestimates the peak values. On the other hand, the Rayleigh probability distribution overestimates the magnitude of troughs to a great extent as is shown in Figure 7. Figure 8 shows a comparison for the peak-to-trough excursions. As can be seen, for the peak-to-trough excursions, the differences between histogram, Rayleigh probability distribution, and the newly-developed probability distribution is less pronounced than that observed for the peaks and troughs, independently.

A comparison of predicted and measured significant values as well as extreme values for peaks, troughs, and peak-to-trough excursions is tabulated in Table 1. It is noted that the predictions are made for the same sea severity as the measured one. Hence, the measured and predicted significant values of the peak-to-trough excursions (significant wave heights) are identical. As can be seen in the table, the predicted values computed by the present theory agree reasonably well with the measured values. However, there is a substantial difference between measured and predicted values computed by applying the Rayleigh probability distribution. It is clear from these results that the probabilistic prediction of peaks and troughs of waves in finite water depth must be based on the concept of the non-Gaussian random process.

CONCLUSIONS

This paper presents a method to statistically predict peaks, troughs, and peak-to-trough excursions of waves in finite water depth such as waves in harbors, bays, and near-shore and offshore seas. The probability density functions are developed based on the concept that waves in finite water depth are a non-Gaussian random process with parameters which depends on the sea severity and water depth.

For predicting statistical characteristics (including extreme values) of waves at a given water depth, a method is developed to identify whether or not waves in the specified sea can be considered as a Gaussian random process at the location. If the sea is considered to be a non-Gaussian random process, then the probabilistic characteristics can be evaluated by applying newly-developed probability density functions.

As an example of application of the present method, numerical computations are carried out for water depth of 8.8 m (28.8 ft) in a sea of significant wave height 3.2 m (10.4 ft). The results of computation show that the newly-developed probability density functions of peaks and troughs both reasonably agree with the histograms constructed from measured data obtained during the ARSLOE Project. The Rayleigh probability density function which is commonly used for predicting waves in deep water substantially underestimates the magnitude of peaks and overestimates the magnitude of troughs for this example.

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ACKNOWLEDGEMENTS

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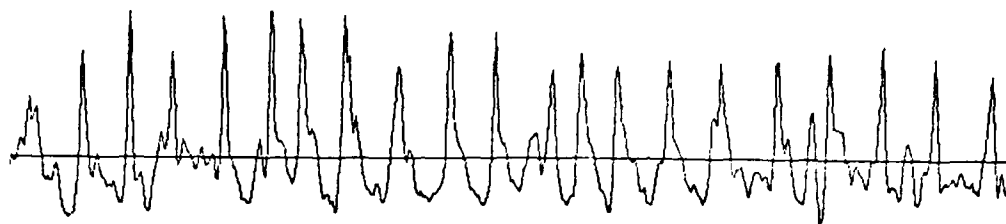
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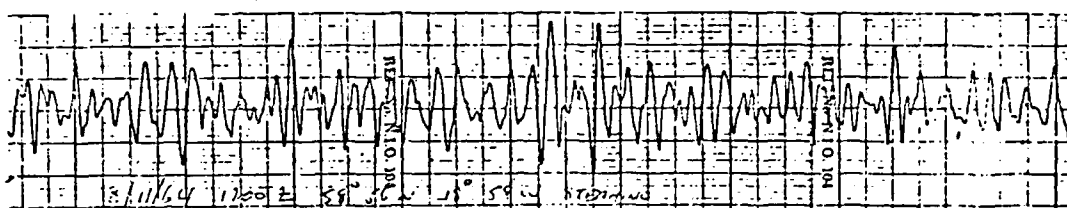
NOMENCLATURE

$f(\ )$	probability density function
$F(\ )$	cumulative distribution function
h	water depth
$H_j(\ )$	Hermite polynomial of degree j
H_s	significant wave height
K	normalization factor
L	normalization factor
m_j	j-th moment of the probability density function
N_+	number of peaks on the positive domain per unit time
$\bar{N}_+$	expected value of N_+
$N_{\xi+}$	number of peaks exceeding a level ξ_+ per unit time
$\bar{N}_{\xi+}$	expected value of $N_{\xi+}$
κ	cumulant
λ_3	scewness
λ_4	kurtosis − 3.0
Ξ, ξ	peak or trough (random variable)
ξ_+	peak
ξ_-	trough
ξ_*	ξ-value for which the probability of exceeding the value is one-third
$\bar{\xi}$	probable extreme value
σ_x^2	variance of wave displacement
σ_{x+}^2	variance which is affiliated with the positive part of waves
σ_{x-}^2	variance which is affiliated with the negative part of waves

Table 1 Comparison of observed and predicted
significant and extreme values

	PEAKS	TROUGHS	PEAK-TO-TROUGH EXCURSIONS
SIGNIFICANT VALUE			
Observed	2.06 m	1.27 m	3.18 m
Predicted by Present theory	2.15	1.34	↓
Rayleigh distribution	1.61	1.61	
EXTREME VALUE			
Observed	3.32 m	1.73 m	5.04 m
Predicted by Present theory	3.79	1.74	4.81
Rayleigh distribution	2.55	2.55	5.11

(a) Coastal waves

(b) Ocean waves

Figure 1 Comparison of coastal and ocean wave profiles

Figure 2 Comparison between observed histogram, Gaussian distribution
(heave line),and non-Gaussian distribution: Water depth 1.4 m,
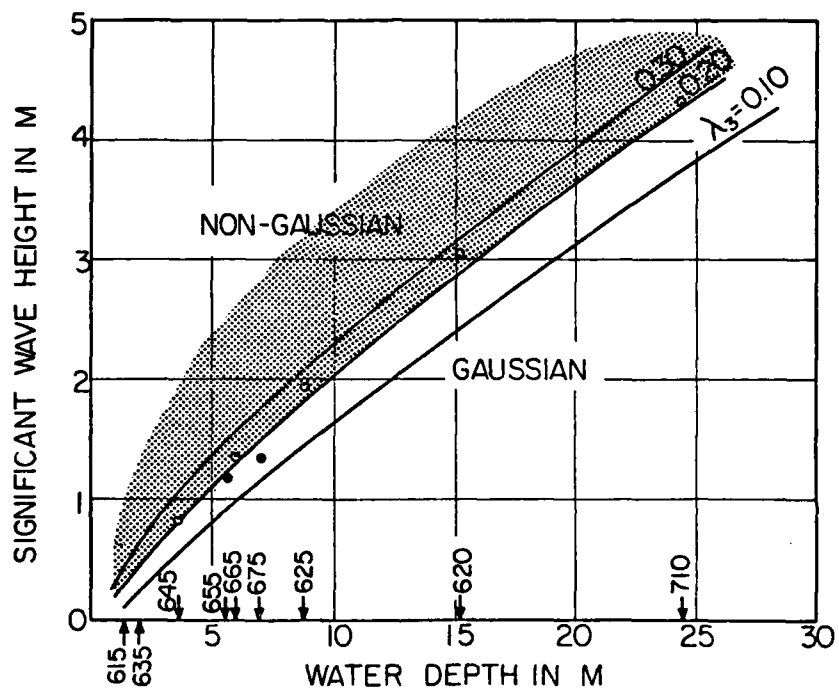significant wave height 2.1 m

Figure 3 Minimum significant wave height above which non-
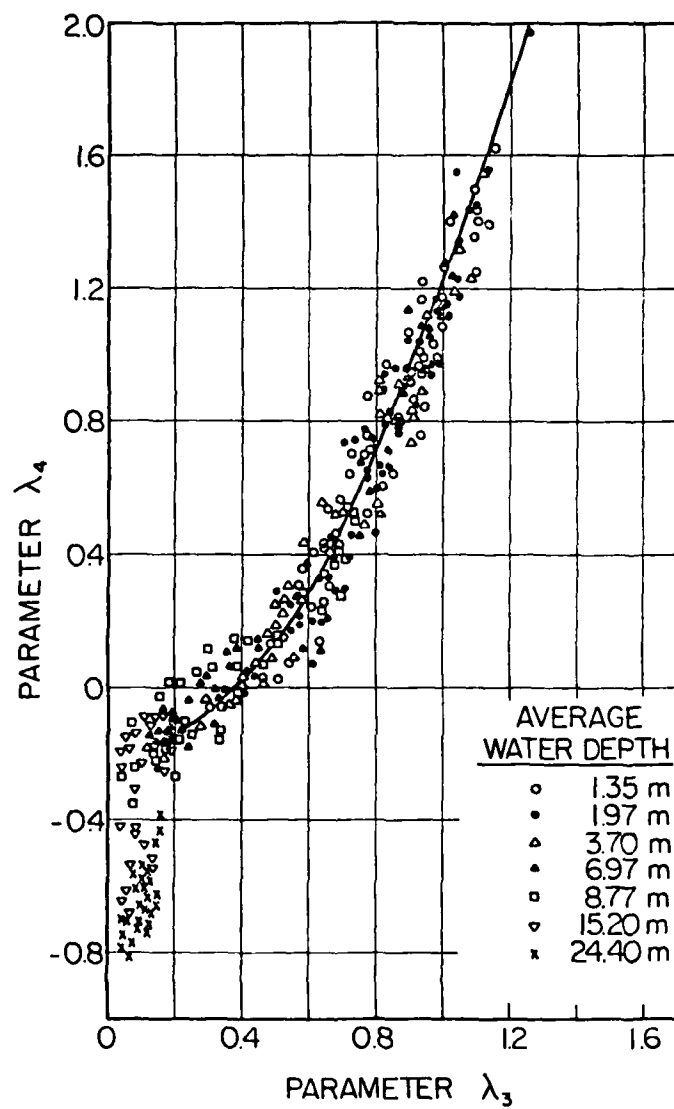Gaussian characteristics are expected as a
function of water depth

Figure 4   Parameter λ_4 as a function of Parameter λ_3 (skewness)
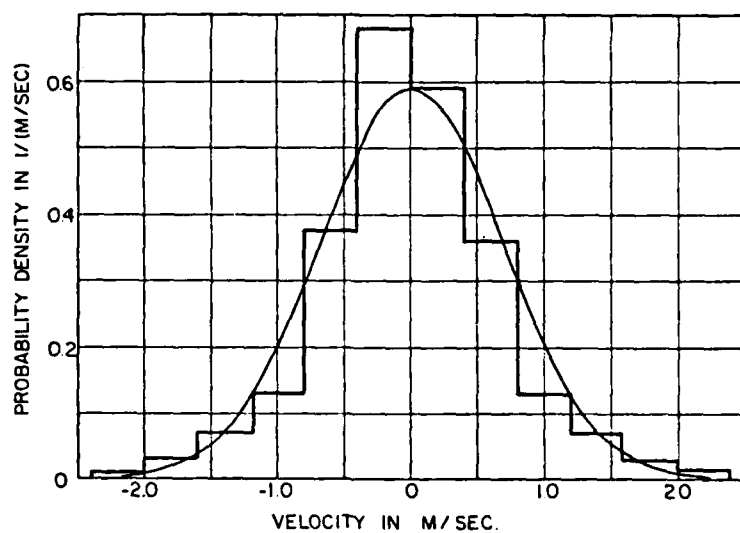
Figure 5   Comparison of wave velocity histogram and Gaussian
           distribution : Water depth 3.7 m, significant wave
           height 2.4 m
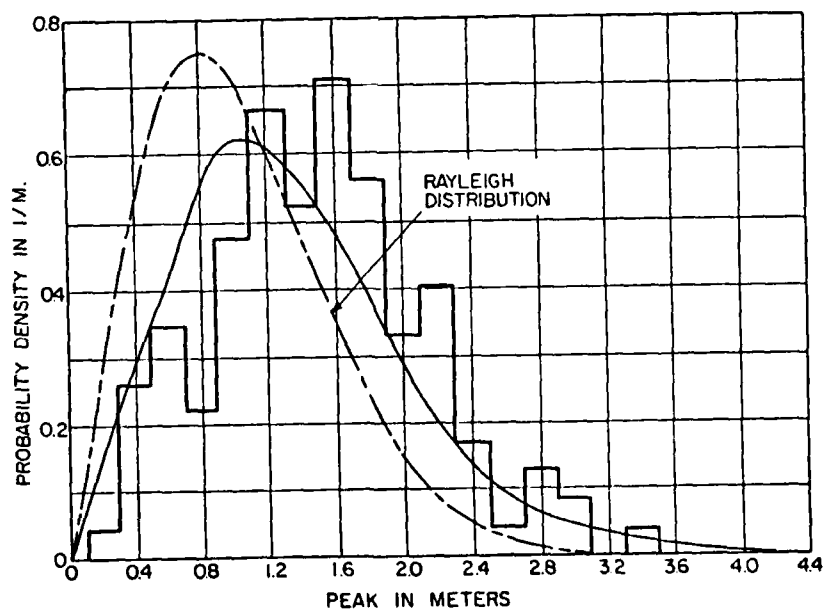

Figure 6   Comparison between observed histogram of peaks, Rayleigh
           distribution, and the newly-developed probability
           distribution: Water depth 8.8 m, significant wave height
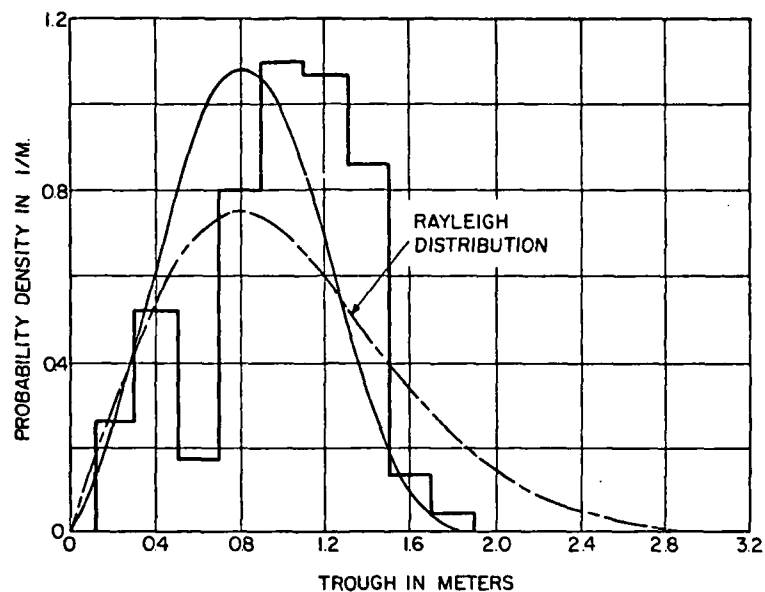   3.2 m

Figure 7   Comparison between observed histogram of troughs,
          Rayleigh distribution, and the newly-developed
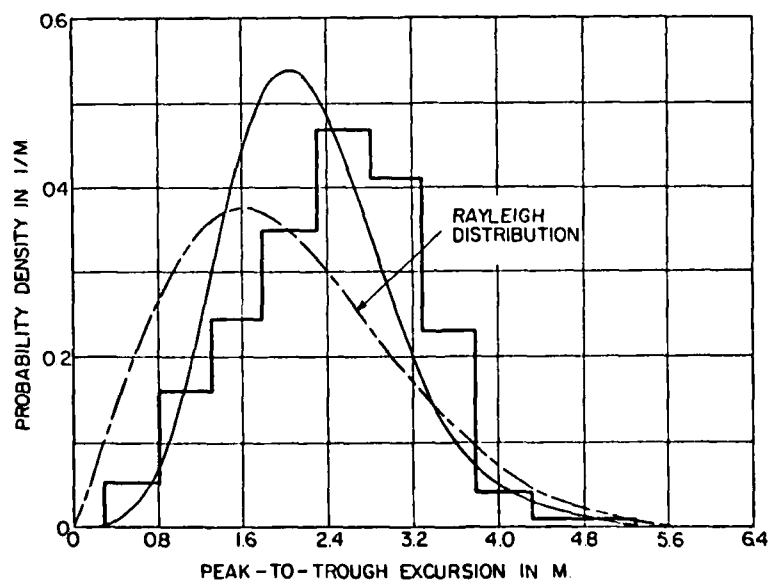   distribution: Water depth 8.8 m, significant wave
          height 3.2 m

Figure 8   Comparison between observed histogram of peak-to-
          trough excursions, Rayleigh distribution, and the
          newly-developed distribution: Water depth 8.8 m,
          significant wave height 3.2 m